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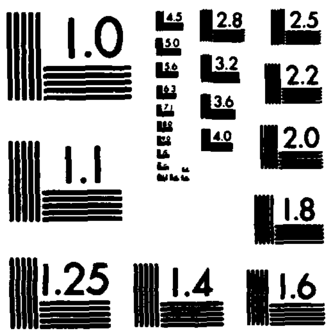

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A mathematical method for inversion in atmospheric remote sensing

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A method is developed for solving Fredholm integral equations of the first kind. The method is particularly intended for use in obtaining atmospheric profiles from remotely sensed radiance measurements and should be generally useful for numerical inversion problems where the solution can be expressed as a linear superposition. Some discussion of general mathematical and physical considerations is also given.

PREFACE

This study was conducted under DA Project 4A161102B52C, Task A, Work Unit 0003, "Inertial Gradiometric and Astronomic Methods for Gravity Field Determination and Point Positioning."

The work was done during Fiscal Year 1983 under the supervision of H.G. Baussus Von Luetzow, Team Leader, Center for Geodesy; and M. Crowell, Jr. and R.D. Leighty, Directors, Research Institute.

COL Edward K. Wintz, CE was Commander and Director and Mr. Walter E. Boge was Technical Director of the U.S. Army Engineer Topographic Laboratories during the report preparation.

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A MATHEMATICAL METHOD FOR INVERSION IN ATMOSPHERIC REMOTE SENSING

INTRODUCTION

The importance of Fredholm integral equations of the first kind is well known to applied mathematicians. One reason for this is that they occur in almost every area of the exact sciences either in the formulation and solution of theoretical problems or in the decomposition of sets of measurements to obtain desirable values and other types of information. Examples of their occurrence include convolution equations, integral transforms, instrumental broadening, and inversion of indirect measurements, in addition to convenience in expressing some physical laws. In areas of remote sensing amenable to quantitative description, the opportunities for their application are particularly numerous. Indeed, for successful solution of many remote sensing problems they are essential and unavoidable; yet they remain unused because of numerical, analytical, and other difficulties, and because researchers in this area often have backgrounds in inexact and descriptive sciences and neither understand nor appreciate the importance of integral equations.

This has not been true in the area of atmospheric inversion, where as early as 1959 Kaplan suggested that it was possible to use remote infrared measurements to infer the structure of the atmosphere. In spite of considerable skepticism, which was based partly on well-founded physical, experimental, and mathematical difficulties, progress has been substantial and a limited degree of success has been achieved.

From an analytical point of view, the difficulty in solving Fredholm equations of the first kind often arises because the equation transforms or maps the manifold on which the unknown functions are defined onto a different, smaller, or more restrictive manifold. In such cases there may no longer be a unique solution because the mapping of one function space onto the other is "many to one." Well-known examples include band-limited convolution products and integral transformations of discontinuous functions using continuous kernels.

For problems where a numerical (not an analytical) type of solution is needed, the integral equation formulation constitutes an incorrect or ill-posed problem in the sense of Hadamard, i.e. a unique solution depending continuously on the transformed or unknown function does not exist. Nevertheless, by redefining the problem, correctness can be established in other ways and solutions can be constructed. In addition to the problem of correctness, severe limitations in the amount of available data, errors in data, and limitations in the numerical methods used, combined with usual computational inaccuracies and possible instability, have led to solutions that were meaningless and even physically impossible for many methods that have been tried.

The method to be proposed here attempts to overcome these difficulties by introducing certain types of discretization and by restricting the function space of the solution. Only the formal aspects are presented and the method is general since it leaves many choices to the ingenuity of a user. It is related to some other methods that have been used, and the discussion should help to elucidate some of the problems encountered in using them.

THE TRANSFER EQUATION

Before proceeding to the mathematical method for its solution, a discussion of the transfer equation will be given. Although this formulation is well known, the brief discussion will define the problem in physical terms and show how the transfer equation is expressed as a Fredholm equation.

It is assumed that the transfer of energy within the atmosphere occurs by radiation through a clear column of air, that clouds are not present, and that scattering processes are unimportant. Furthermore, a local condition of radiative thermodynamic equilibrium is assumed so that the ratio of the coefficient of emission to that of absorption is given by the Kirchhoff-Planck equation

$$B(\nu,\ T) = \frac{2h\nu^3}{C^2} \frac{1}{e^{\frac{k\nu}{kT}} -1} \tag{1}$$

where ν is the frequency of the radiation and T is the absolute temperature. This represents a convenient abstraction for situations where continuous spectra are encountered and where no correlation exists between the particular emissions and absorptions that participate in the multiplicity of processes involved.

If p is used for pressure level, the temperature profile is defined by T(p) corresponding with measured radiance intensity $I(\nu)$, and an initial approximation is assumed to be given by $\bar{T}(p)$, $\bar{I}(\nu)$. The quantity

$$\Delta\ I(\nu) = I(\nu) - \bar{I}(\nu) \tag{2}$$

is then expressable by a different form of the radiative transfer equation

$$\Delta I(\nu) = \delta B(s)\ \tau_\nu(s) + \int_{\tau_{\nu(s)}}^{1} \Delta B(p)\ d\tau_\nu(p) \tag{3}$$

where $\tau_\nu(p)$ is the transmittance of the atmosphere from pressure level p to the top and where s refers to the surface pressure level. Furthermore, δB and ΔB are given by

$$\delta B(s) = B(\nu_r, \hat{T}(p)) - B(\nu_r, \bar{T}(s)) \tag{4}$$

$$\Delta B(s) = B(\nu_r, T(p)) - B(\nu_r, \bar{T}(p)) \tag{5}$$

where $\hat{T}(p)$ is the assumed surface temperature and has been a particular source of difficulty. By noting that

$$\int_{\tau_\nu(s)}^{1} d\tau_\nu(p) = 1 - \tau_\nu(s) \tag{6}$$

both terms of the transfer equation (3) can be combined.

$$\Delta I(\nu) - \int_{\tau_{\nu(s)}}^{1} \left\{\Delta B(p) + \delta B(s)\left[\frac{\tau_\nu(s)}{1-\tau_\nu(s)}\right]\right\} d\tau_\nu(p) \tag{7}$$

It is to be expected that a correspondence (not necessarily one to one) exists between the bracketed term and some function of the pressure profile

$$h(p) = \frac{\tau_\nu(s)}{1 - \tau_\nu(s)} \tag{8}$$

so that the transfer equation assumes the customary form for this problem

$$\Delta I(\nu) = -\int_{0}^{s} x(p) \frac{d\tau_\nu(p)}{dp} dp \tag{9}$$

where

$$x(p) = \Delta B(p) + \delta B(s)\, h(p) \tag{10}$$

In this form, the radiative transfer equation (9) is a Fredholm integral equation of the first kind to be solved for x(p) where the boundary term $\delta B(s)\, h(p)$ is to be subtracted to obtain $\Delta B(p)$. Since the function h(p) is not known, it could be constructed by integrating the transfer equation for many known widely divergent profiles at several wavelengths and using

$$h(p) = [x(p) - \Delta B(p)]/\delta B(s) \tag{11}$$

but it more often must be obtained by using surface transmittances. Different functions h(p) are characteristic of atmospheres of different temperature and humidity type.

THE MATHEMATICAL INVERSION METHOD

In more conventional mathematical notation, the equation of transfer (9) can be written in the form

$$f(x) = \int_a^b K(x, \xi)\, u(\xi)\, d\xi \tag{12}$$

where the function f is given by a series of measurements for fixed values of x and where the kernel K is considered to be known. The problem of inversion consists of recovering the unknown function u to within some reasonable limits. In atmospheric inversion problems, the kernel function is smooth and limits the ability to recover higher frequency components resulting in an ambiguity (or in some methods, a so-called "inherent instability") in the inversion for f.

In order to effect an inversion, a set of basis functions $u_i(x)$ is selected to span a function space of low dimension, but capable of giving a representation of the solution u(x) in the form

$$u(x) = \sum_{i=1}^{N} a_i u_i(x) \tag{13}$$

This is suggested by the linearity of the integral equation (12), but the basis functions need not physically represent actual atmospheric profiles. They need only be capable of being summed to give a good approximation to the profile being sought. The solution will consist of determining the constants a_i.

From the basis functions u_i, a new set of N functions $f_i(x)$ is to be computed by performing the quadratures indicated by

$$f_i(x) = \int_a^b K(x, \xi)\, u_i(\xi)\, d\xi \tag{14}$$

for the values of x at which measurements have been made.

Another set of N functions $\phi_j(x)$ is also to be constructed, at the same values of the argument x, as linear combinations of the functions $f_i(x)$

$$\phi_j(x) = \sum_{k=1}^{N} \alpha_{jk} \; f_k(x) \tag{15}$$

where the constants α_{jk} are determined by imposing the condition

$$\int_a^b \phi_j(x) \; f_\ell(x) \; dx = \delta_{j\ell} \tag{16}$$

where the integral implies a numerical quadrature over the appropriate values of x. Multiplying both sides of equation (15) by $f_\ell(x)$ and performing a similar quadrature leads to

$$\int_a^b \phi_j(x) \; f_\ell(x) \; dx = \sum_{k=1}^{N} \alpha_{jk} \int_a^b f_k(x) \; f_\ell(x) \; dx \tag{17}$$

which becomes

$$\phi_{j\ell} = \sum_{k=1}^{N} \alpha_{jk} \; \beta_{k\ell} \tag{18}$$

when equation (16) is introduced and the constants $\beta_{k\ell}$ are defined by equation (19).

$$\beta_{k\ell} = \int_a^b f_k(x) \; f_\ell(x) \; dx \tag{19}$$

The quantities $\beta_{k\ell}$ are compatible by the numerical quadrature indicated in equation (19) since the functions $f_k(x)$ are known. Then, α_{jk} satisfying equation (18) can be found by taking the matrix α_{jk} to be the inverse of $\{\beta_{jk}\}$ where it may be convenient to make use of the fact that both matrices must be symmetrical. The functions $\phi_j(x)$ can now be obtained by use of equation (15). It will be found that they are not needed for a final computation, but are useful in the derivation of the solution.

Inserting equation (13) into the integral equation (12),

$$f(x) = \int_a^b K(x, \xi) \sum_{m=1}^{N} a_m u_m(\xi) \, d\xi \tag{20}$$

$$f(x) = \sum_{m=1}^{N} a_m \int_a^b K(x, \xi) u_m(\xi) \, d\xi \tag{21}$$

it is found by using equation (14) that

$$f(x) = \sum_{m=1}^{N} a_m f_m(x). \tag{22}$$

Multiplying both sides of equation (22) by $\phi_n(x)$ and integrating (performing a quadrature) yields a method for computing the coefficients a_n.

$$\int_a^b \phi_n(x) f(x) \, dx = \sum_{m=1}^{N} a_m \int_a^b \phi_n(x) f_m(x) \, dx \tag{23}$$

$$= \sum_{m=1}^{N} a_m \delta_{nm} \tag{24}$$

$$\int_a^b \phi_n(x) f(x) \, dx = a_n \tag{25}$$

The insertion of equation (15) for $\phi_n(x)$ into equation (25) leads to the proper expression for computing the coefficients a_i in equation (13),

$$a_n = \int_a^b \sum_{k=1}^{N} \alpha_{nk} f_k(x) f(x) \, dx \tag{26}$$

$$a_n = \sum_{k=1}^{N} \alpha_{nk} \int_a^b f_k(x) f(x) \, dx \tag{27}$$

$$a_n = \sum_{k=1}^{N} \alpha_{nk} \gamma_k \tag{28}$$

where γ_k is defined as follows.

$$\gamma_k = \int_a^b f_k(x)\, f(x)\, dx \tag{29}$$

The final solution can now be written by replacing a_i in equation (13) with equation (28).

$$u(x) = \sum_{n=1}^{N} \sum_{k=1}^{N} \alpha_{nk} \gamma_k u_n(x) \tag{30}$$

Recapitulating, the steps in finding the solution consist of performing the quadratures indicated in equations (14), (19), and (29) and inverting $\{\beta_{k\ell}\}$ to obtain $\{\alpha_{k\ell}\}$.

It should be remarked that the same type of quadrature formula can be used for most cases involved. Also, N must not exceed the number of points at which the function f(x) is given if the matrix $\{\beta_{n\ell}\}$ is to be successfully inverted.

CONCLUSION

A method of solving Fredholm integral equations of the first kind for use in the numerical inversion of remotely sensed radiance measurements has been presented. Use of this method requires the specification of a basic set of functions from which the solution can be written as a linear superposition. It does not impose further unnecessary conditions, such as the orthogonality often used in obtaining such solutions. This should allow a good representation among the physically realizable solutions from a relatively small number of basis functions. This is important since the information available for the function to be inverted is usually severely limited by physical restrictions on the number and types of measurements.

The general mathematical aspects of the problem and a brief derivation of the mathematical equation from physical considerations have been presented.

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